Honors - Economic Analysis III

Lecture 11: Incomplete markets

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This lecture

- Complete markets = Complete insurance against idiosyncratic risk

- Three of many big questions:

1. How close to full insurance can people get with simple, realistic assets?

2. What are the implications of idiosyncratic risk for interest rates?

3. If we have less than full insurance, then can we establish a framework for thinking about the welfare gains from *redistributive* policy?

- If interested in these issues, take Prof. Kaplan's upper level course!

Hugget (1993)

- Equity premium puzzle: Large equity premium ($6\% \approx 7\% - 1\%$)
- *Question*: Why has the real risk-free rate been less than one percent?
- Mehra Prescott (1989) - Rep. household implicitly assumes *complete markets* (As we learned at the end of class on Tuesday). The only risk that is priced is aggregate risk.
- What if we make markets incomplete? No arrow securities. One risk-free bond.
- *Idea*: Maybe large idiosyncratic risk with *incomplete insurance* leads to a high demand for the limited set of safe assets?
- This would increase their price, and push down their rate of return.

Hugget (1993) - Environment

- *Time* - Discrete $t = 0, 1, 2\dots$
- *Agents* - Large set of individuals $i \in \{1, 2, \dots, N\}$
- *States* - Economy can be in finite states $s \in \{s_1, \dots, s_S\}$. Prob $\pi(s^t)$

  Assume that s_t is *Markov* such that $\pi(s_{t+1}|s^t) = \pi(s_{t+1}|s_t)$.
- *Endowments* - Each individual's finite endowment depends only on s_t: $y^i(s_t) \in [0, \infty)$
- *Goods* - One perishable consumption good
- *Preferences* - Utility of each individual household at date 0 is

$$U^i(s_0) = \sum_{t=0}^{\infty} \sum_{s^t|s_0} \beta^t \pi(s^t) u^i(c_t^i(s^t)) \quad , \quad \beta \in (0, 1)$$

Hugget (1993) - Environment

- *Time* - Discrete $t = 0, 1, 2 \dots$
- *Agents* - Large set of individuals $i \in [0, 1]$
- *Endowments* - Each individual's finite endowment $y_{it} \in [0, \infty)$

  Assume that y_{it} is *Markov* with probabilities $y_{it+1} \sim \pi_y(y'|y_{it})$.

  Assume $\int_0^1 y_{it} di = Y$ is constant. No aggregate risk
- *Goods* - One perishable consumption good
- *Preferences* - Utility of each individual household at date 0 is

$$U_0^i = \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t u(c_{it}) \right] \quad , \quad \beta \in (0, 1)$$

Assets

- Individuals can only trade a *risk-free bond*
- The bond has a price $Q < 1$, giving the risk-free return $R = 1/Q$.
- Since there is no aggregate risk, this is constant.
- Individual budget constraint

$$c_{it} + Qb_{it+1} \leq y_{it} + b_{it}$$

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- *Borrowing limit*: Let individuals *borrow* up to some exogenous borrowing limit

$$b_{it+1} \geq -\phi$$

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- *Borrowing limit*: Let individuals *borrow* up to some exogenous borrowing limit

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- In equilibrium it must be the case that bonds are in zero net supply

$$\int_0^1 b_{it} = 0$$

Recursive competitive equilibrium

- A *recursive competitive equilibrium* is
- A value function $v(b,y)$, a demand for bonds $b'(b,y)$, a bond price Q, and a *distribution* of individuals $h(b,y)$. (i.e. gives the mass $h(b,y) \geq 0$ of individuals with (b,y), and $\sum_{b,y} h(b,y) = 1$)

  such that
- *Optimality* - Given the price Q, the value function $v(b,y)$ and demand for bonds $b'(b,y)$ solve

$$v(b,y) = \max_{b'} \; u(c) + \beta \sum_{y'} \pi_y(y'|y) v(b',y')$$

  subject to

$$c + Qb' \leq y + b \qquad , \qquad b' \geq -\phi$$

- *Market clearing* - Markets clear for bonds and goods:

$$\sum_{b,y} b'(b,y)\, h(b,y) = 0 \qquad , \qquad \sum_{b,y} c(b,y)\, h(b,y) = Y$$

- *Consistency* - The distribution of individuals $h(b,y)$ is consistent with $b'(b,y)$ and $\pi_y(y'|y)$:

$$h(b',y') = \sum_{b,y} \underbrace{\mathbf{1}[b' = b'(b,y)]\pi_y(y'|y)}_{\text{Fraction of } h(b,y) \text{ that go to } (b',y')} \; h(b,y)$$

Hugget (1993) - Calibration

- Model period is two months
- Think of y_{it} due to either employed or unemployed: $y_{it} \in \{y_U, y_E\}$
- Normalize $y_E = 1$, and $y_U = 0.10$
- Calibrate $\pi_y(y'|y)$ to unemployment durations and probability of job loss
- Goes to *individual level data* and computes a 3 percent chance of losing job in a given month

$$\pi_y(y_U|y_E) = 0.075$$

- Average duration of unemployment is 17 weeks, which is 2×2 months

$$\pi_y(y_E|y_U) = 0.50$$

- Set β such that annual $\widetilde{\beta} = \beta^{1/12} = 0.96 \rightarrow Q = \beta \rightarrow r = \frac{1-Q}{Q} = \mathbf{0.04}$ under complete markets
- Set $u(c) = c^{1-\sigma}/(1-\sigma)$, with $\sigma = 1.5$ to be the same as Mehra-Prescott
- Considers a range of values of credit limit

$$\phi \in \{8, 6, 4, 2\} \qquad \text{Equivalent to } \{1.50, 1.13, 0.75, 0.38\} \text{ times average annual income}$$

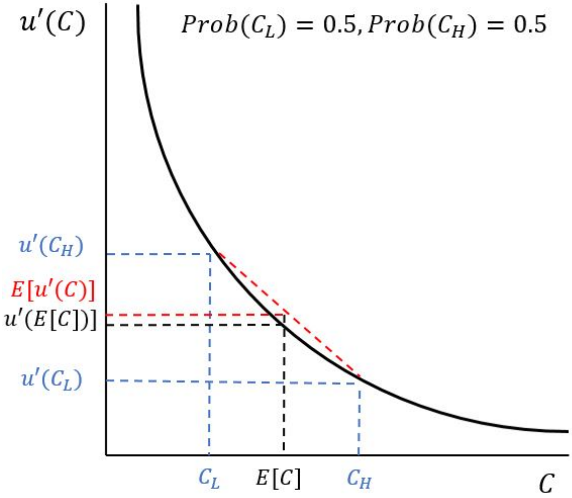
Average year gives 10.6 months employed, so income of 5.3

Savings

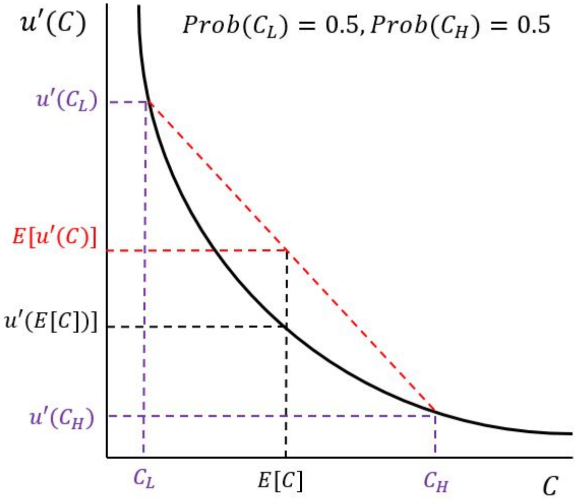
- Individuals save for two reasons

1. Precautionary motives we have discussed

Precautionary saving and idiosyncratic risk



Precautionary saving and idiosyncratic risk



Savings

- Individuals save for two reasons

1. Precautionary motives we have discussed
2. Fear of being credit constrained

- When employed
  - Accumulate assets such that $b_{it+1} > b_{it}$ up to some point
- When unemployed
  - Decumulate assets such that $b_{it+1} < b_{it}$
  - If you remain unlukcy and unemployed then at some point hit borrowing limit:

$$c_{it} = y_U$$

Effect of credit limit - ϕ

Table 1

Coefficient of relative risk aversion $\sigma = 1.5$.

Credit limit ($\underline{a}$)	Interest rate (r)
-2	-7.1%
-4	2.3%
-6	3.4%
-8	4.0%

- As credit limit becomes tighter (closer to zero), the interest rate falls. Even goes negative!

- With a *slack* credit limit (e.g. $\phi = 8$), individuals have less demand for assets
- Lowers their 'price' in equilibrium $\downarrow Q$, increasing their return $\uparrow r$
- If can borrow up to 1.5 times average annual earnings, then get all the way to *complete markets* interest rate

  ... without any arrow securities

Effect of credit limit - ϕ

Table 1

Coefficient of relative risk aversion $\sigma = 1.5$.

Credit limit ($\underline{a}$)	Interest rate (r)
-2	-7.1%
-4	2.3%
-6	3.4%
-8	4.0%

- With a *tight* credit limits (e.g. $\phi = 2$), individuals have *more* demand for assets
- Increases their 'price' in equilibrium $\uparrow Q$, decreasing their return $\downarrow r$
- If can borrow only one third of average annual earnings, then have *negative real rates*
- Inconceivable with complete markets, becomes very plausible with incomplete markets

Effect of credit limit - ϕ

Table 2

Coefficient of relative risk aversion $\sigma = 3.0$.

Credit limit ($\underline{a}$)	Interest rate (r)
-2	$-23\ \%$
-4	-2.6%
-6	1.8%
-8	3.7%

- With higher risk aversion, effects are amplified

- Note that this is only in the case where insurance against downside risk is taken away!

- With lots of borrowing, can still get close to complete markets

Corresponding representative agent economy

- Recall average endowment was $\mathbb{E}[y_{it}] = 5.3$ in a year
- Give this to a representative agent $Y = 5.3$
- Then to hold zero bonds, requires Euler equation to hold

$$Qu'(C) = \beta u'(C')$$

- Implies interest rate

$$1 + r = 1/Q = 1/\beta = 1.042$$

- Very different to the complete markets case where $r = 4.2\%$!

- Results - Can prove that $(1 + r) < \beta$ in equilibrium

Kaplan Violante (2010)

- How much consumption insurance do individuals achieve under incomplete markets?
- Income: Systematic age component θ_t^{age}, transitory shocks ε_{it}, persistent shocks η_{it}

$$y_{it} = \theta_t^{age} + z_{it} + \varepsilon_{it} \qquad \varepsilon_{it} \sim N(0, \sigma_\varepsilon)$$

$$z_{it} = \rho_z z_{it-1} + \eta_{it} \qquad \eta_{it} \sim N(0, \sigma_\eta)$$

- Estimate this using microdata: household income data, gives a rich model of $\pi_y(y'|y)$
- Add realistic taxes, social security, borrowing limits.
- Same as Huggett (1993), this implies a constant *aggregate endowment* of $Y = 1$

Check - Does the model give realistic lifecycle dynamics?

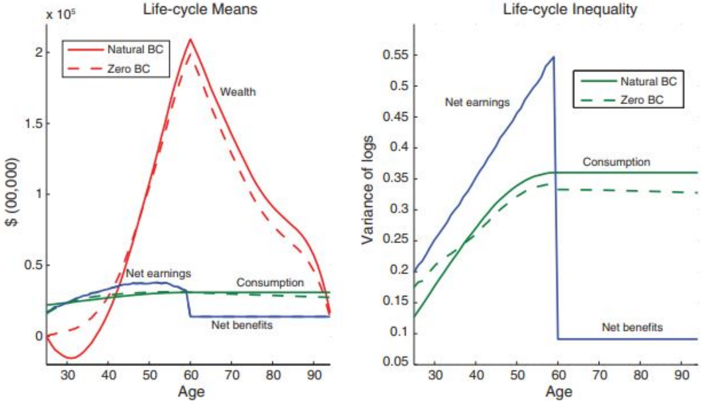


FIGURE 1. LIFE-CYCLE PROFILES FOR MEANS AND VARIANCES IN THE NBC AND ZBC ECONOMIES

How much consumption insurance do individuals have?

- The *insurance coefficient* answers this question:

$$\phi^x = 1 - \frac{\Delta \log c_{it}}{\Delta \log y_{it}} \qquad \text{(With complete markets = 1 as } c_{it} = \tilde{\theta}^i C\text{)}$$

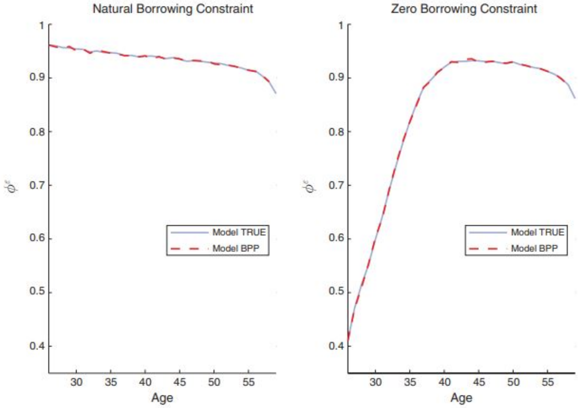


FIGURE 2. AGE PROFILES OF INSURANCE COEFFICIENTS FOR TRANSITORY SHOCKS