Honors - Economic Analysis III

Lecture 9: RBC Model

Simon Mongey

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This lecture

- Review one period labor supply model with a change in productivity A
- Review business cycle facts
- Study Real Business Cycle model
- PS5 - Labor supply / RBC model
- Next - Asset pricing

One-period labor supply problem - Increase in A

- Labor market equilibrium

$$\frac{v'(N)}{u'(C)} = \frac{W}{P} = AF_N(\overline{K}, N)$$

- Suppose there is an increase in A, so that labor demand increases

- What happens to N in equilibrium?

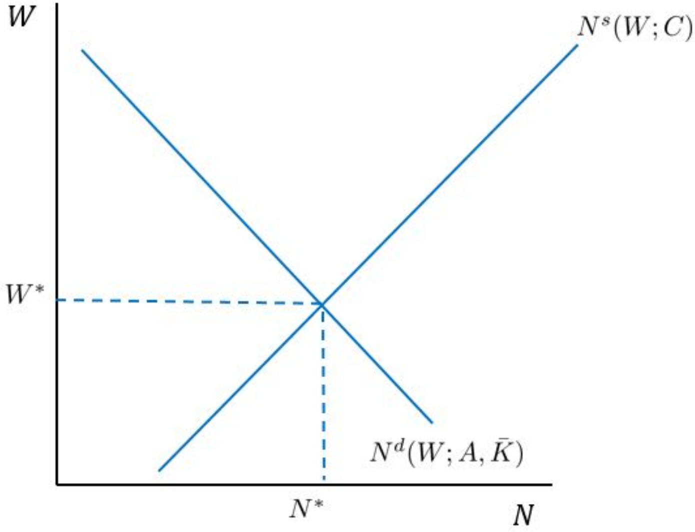
- Depends on how the marginal utility of consumption changes in equilibrium

  1. If $u'(C)$ *decreases a lot* ($\uparrow\uparrow C$), then N will *decrease*

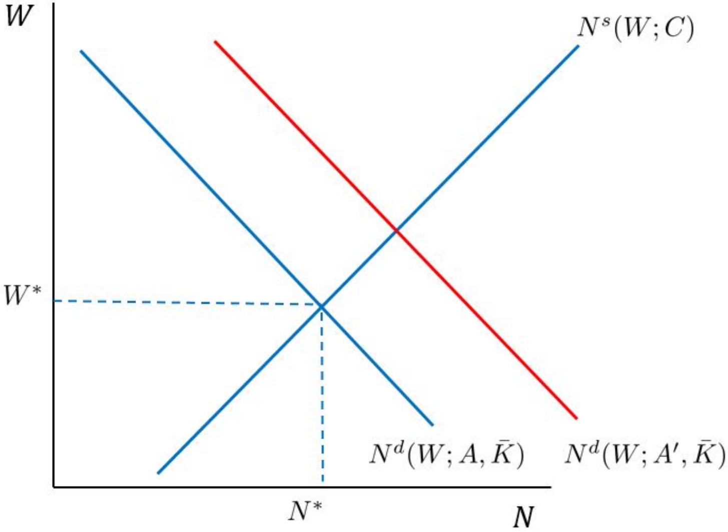
  2. If $u'(C)$ *decreases a little* ($\uparrow C$), then N will *increase*

  3. Intermediate case where then N does not change

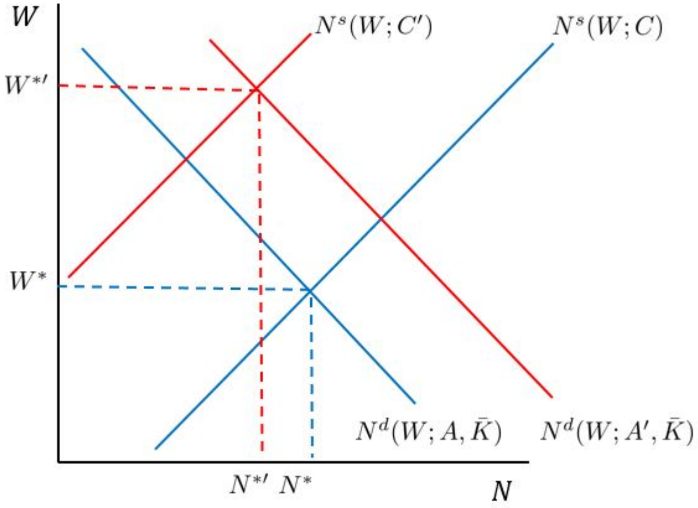
Labor market equilibrium



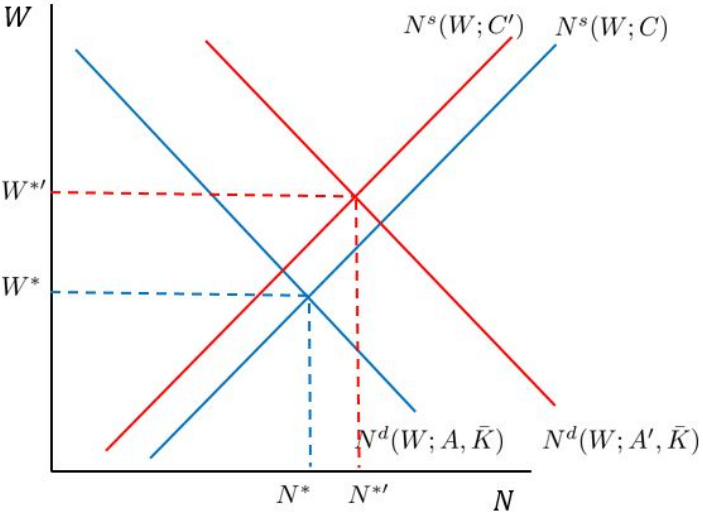
Labor market equilibrium - $\uparrow A$



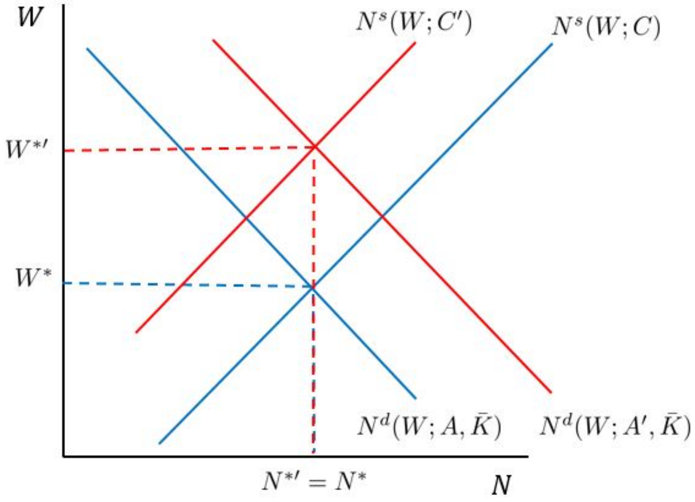
Labor market equilibrium - $\downarrow\downarrow\downarrow u'(C)$ - Income effect



Labor market equilibrium - $\downarrow u'(C)$ - Sub'n effect



Labor market equilibrium - $\downarrow\downarrow C$ - Net out



One-period labor supply problem

- Let $U(C,N)$ be given by

$$U(C,N) = \log C + \phi \frac{N^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}}$$

- Full set of equilibrium conditions

$$\begin{aligned} \uparrow AF(\overline{K}, N) &= \uparrow C \\ \uparrow AF_N(\overline{K}, N) &= \uparrow \frac{W}{P} \\ \phi N^{\frac{1}{\varphi}} \uparrow C &= \uparrow \frac{W}{P} \end{aligned}$$

- Output increases, therefore C increases **by same amount as** Y
- Ambiguous effect on N

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$$\uparrow AF_N(\overline{K}, N) \quad = \quad \phi N^{\frac{1}{\varphi}} \uparrow AF(\overline{K}, N)$$

- Output increases, therefore C increases **by same amount as Y**
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One-period labor supply problem

- Let $U(C, N)$ be given by

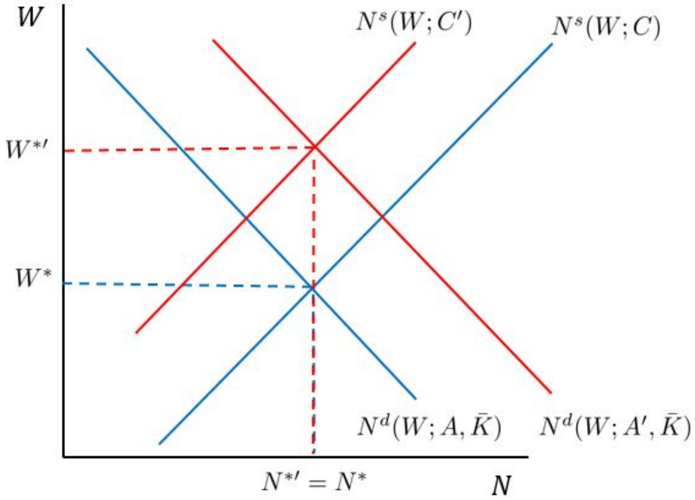
$$U(C,N) = \log C + \phi \frac{N^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}}$$

- Full set of equilibrium conditions

$$F_N(\overline{K}, N) \quad = \quad \phi N^{\frac{1}{\varphi}} F(\overline{K}, N)$$

- Equilibrium $\uparrow C$ increases $\uparrow (1/u'(C))$ one-for-one
- This reduces N supply to exactly offset the increase in N demand

Labor market equilibrium



One-period labor supply problem

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One-period labor supply problem

- Let $U(C,N)$ be given by

$$U(C,N) = \log C + \phi \frac{N^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}}$$

- Full set of equilibrium conditions

$$\frac{F_N(\overline{K},N)}{F(\overline{K},N)} = \phi N^{\frac{1}{\varphi}}$$

- Equilibrium $\uparrow C$ reduces $1/u'(C)$ one-for-one
- This reduces N supply to exactly offset the increase in N demand

One-period labor supply problem

- Let $U(C,N)$ be given by

$$U(C,N) = \log C + \phi \frac{N^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}}$$

- Full set of equilibrium conditions

$$\underbrace{\frac{F_N(\overline{K},N)N}{F(\overline{K},N)}}_{\alpha} = \phi N^{\frac{1+\varphi}{\varphi}}$$

- Equilibrium $\uparrow C$ reduces $1/u'(C)$ one-for-one
- This reduces N supply to exactly offset the increase in N demand

One-period labor supply problem

- Let $U(C,N)$ be given by

$$U(C,N) = \log C + \phi \frac{N^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}}$$

- Think about $\overline{K}$ as some *steady-state* level of capital
- For different values of A, steady-state $\overline{K}(A)$ adjusts.
- However, labor supply is constant:

$$\alpha = \phi N^{\frac{1+\varphi}{\varphi}}$$

- Result - *Under these preferences equilibrium labor supply is constant with respect to* ***permanent*** *changes in productivity A*

One-period labor supply problem

- Let $U(C,N)$ be given by

$$U(C,N) = \log C + \phi \frac{N^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}}$$

- Result - *Under these preferences equilibrium labor supply is constant with respect to* ***permanent*** *changes in productivity A*
- We can think of these as desirable *long-run* properties of the model we will now study
- Therefore, we will adopt these preferences in the RBC model
- The key question will then be ...
- *Can we get changes in the* ***short-run*** *in employment, output, consumption that look like U.S. business cycles from* ***short-run*** *changes in A?*

Environment - Centralized , Stochastic

- *Time* - Discrete $t = 0, 1, 2 \dots$
- *Agents* - Representative household
- *Goods* - One good can either be used for consumption or investment

$$C_t + I_t = Y_t$$

- *Endowments* - Household owns capital K_0 and has $\overline{L}$ units of labor
- *Preferences* - Utility of the household at date 0 is

$$\mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \left\{ \log C_t + \phi \frac{N_t^{1+\frac{1}{\psi}}}{1+\frac{1}{\psi}} \right\} \right] \quad , \quad \beta \in (0,1)$$

- *Technology* - Constant returns to scale production technology $Y_t = A_t F(K_t, N_t)$, where given A_0, $\{A_t\}_{t=1}^{\infty}$ is given by

$$\log A_{t+1} = (1-\rho) \log \overline{A} + \rho \log A_t + \underbrace{\varepsilon_{t+1}}_{\textbf{`Shock'}} \quad , \quad \varepsilon_{t+1} \sim N(0, \sigma_\varepsilon)$$

Capital depreciates at rate δ

Why?

Kydland and Prescott (1982)

- In the data we observe large, persistent, fluctuations in A_t

$$\begin{aligned} Y_t &= A_t K_t^{\alpha} N_t^{1-\alpha} \\ \log Y_t &= \alpha \log K_t + (1-\alpha) \log N_t + \log A_t . \end{aligned}$$

- **Q**: Can 'shocks' to A_t account for business cycles in the NC model?

Why do we care?

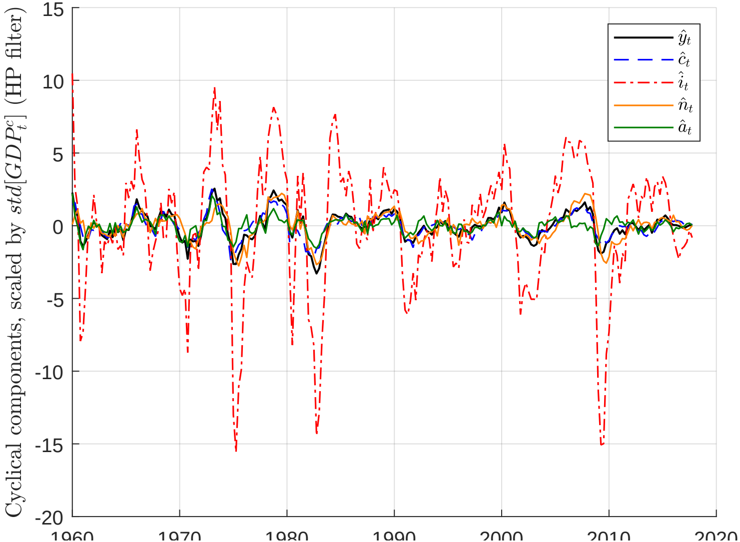
- NC model: (i) *Always in equilibrium*, (ii) *Welfare theorems hold*

- 1970s 'Keynesian' view: Recessions represent the economy being 'out of equilibrium' in some way. Government can intervene.

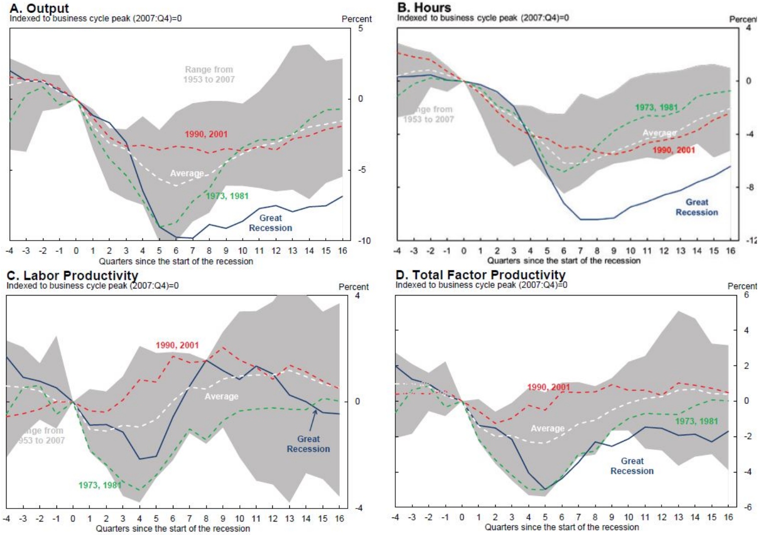
- Normative - If the answer is 'yes' then recessions are the economy's natural responses to exogenous shocks

- Positive - We have a quantitative theory of business cycles!

Business cycles - US data



Business cycles - US data



Fernald (2014) - Productivity and Potential Output Before, During, and After the Great Recession -

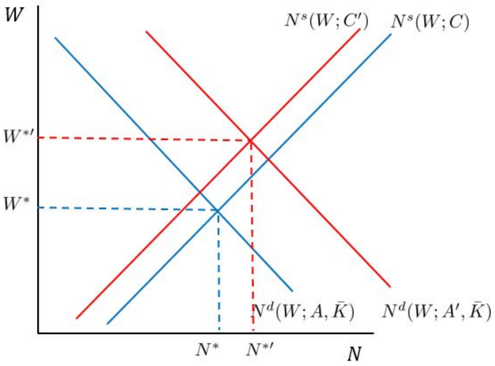
Business cycles - US data

		Volatility $std[\hat{x}_t]/std[\hat{y}_t]$	Covariance $corr(\hat{x}_t, \hat{y}_t)$	Persistence $corr(\hat{x}_t, \hat{x}_{t-1})$
Output (GDP)	$\hat{y}_t$	1	1	0.86
Consumption	$\hat{c}_t$	0.81	0.87	0.87
Investment	$\hat{i}_t$	4.52	0.90	0.83
Hours	$\hat{n}_t$	0.98	0.84	0.86

- Consumption - Less volatile than output
- Investment - Much volatile than output
- Hours - As volatile as output
- All series highly correlated with output

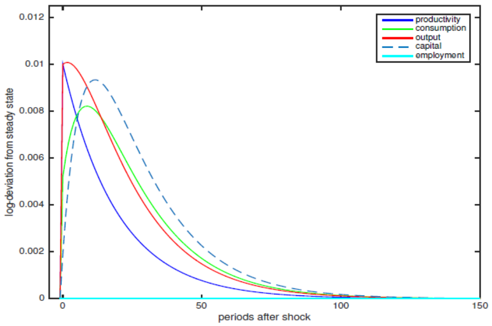
Recall effect of $\uparrow G$ in labor supply model: $\uparrow Y, \uparrow N, \downarrow C$

Key mechanism - Labor market equilibrium



- Due to *consumption smoothing* C does not increase by same amount as Y
- Then $u'(C)$ does not decrease as much
- So labor supply does not contract as much
- Result - Equilibrium employment increases

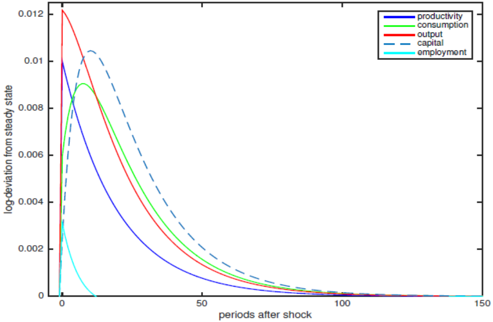
Impulse responses - $\varphi \to 0$



- As $\varphi \to 0$, then just supply all labor all the time: $N_t = 1 - \overline{L}$

$$U(C_t, N_t) = u(C_t) - \phi \frac{N_t^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}} \quad \to \quad U(C_t, N_t) = u(C_t) - \phi N_t$$

Labor market equilibrium - $\varphi > 0$

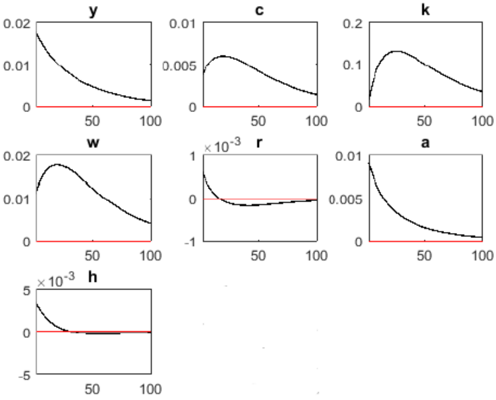


- Recall that the labor supply function is

$$\phi C_t N_t^{\frac{1}{\varphi}} = W_t \quad \rightarrow \quad N_t = u'(C_t) \times W_t^{\varphi}$$

- Consumption responds *less than output*. Labor supply shifts outwards.
- This increases the response of output

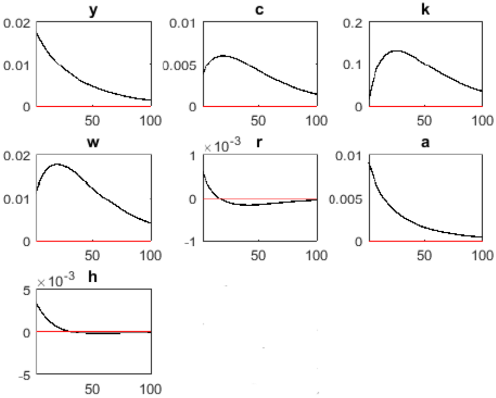
Response of economy to a shock to A_t



1. While the interest rate is above steady-state, consumption is *increasing*

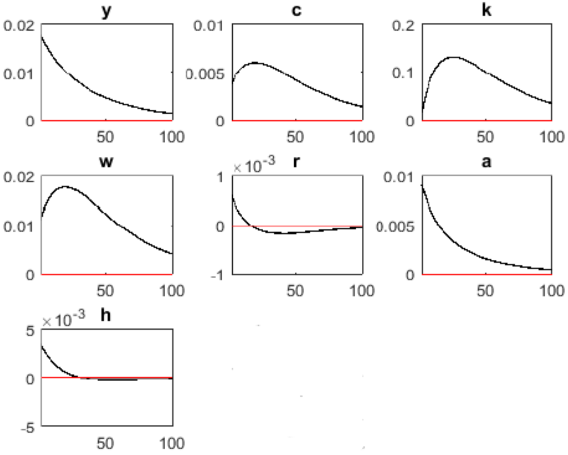
$$1 = \beta\left[(1+\bar{r})\right] \quad \rightarrow \quad \frac{1}{C_t} = \beta \mathbb{E}_t\left[\frac{1}{C_{t+1}}(1+r_{t+1})\right]$$

Response of economy to a shock to A_t



2. Consumption increases less than one-for-one with Y, so labor increases

Response of economy to a shock to A_t



3. Path for consumption leads K_t to accumulate. Decumulates as R_t starts again increasing.

Kydland and Prescott's Quantitative Results

TABLE III

MODEL'S STANDARD DEVIATIONS AND CORRELATIONS WITH REAL OUTPUT[a]

Variable	Standard Deviations: Means (Standard Deviations) of Sample Distribution[b]	Correlations with Output: Means (Standard Deviations) of Sample Distribution
Real Output	1.80 (.23)	—
Consumption	.63 (.09)	.94 (.01)
Investment	6.45 (.62)	.80 (.04)
Inventories	.89 (.06)	−.15 (.11)
Inventories plus	2.00 (.20)	.39 (.06)
Capital Stock	.63 (.08)	−.07 (.06)
Hours	1.05 (.13)	.93 (.01)
Productivity	.90 (.10)	.90 (.02)
Real Interest Rate (Annual)	.23 (.02)	.47 (.10)

[a]The length of the sample period both for the model and for the U.S. economy is 118 quarters.
[b]Measured in per cent.

TABLE IV

SAMPLE STANDARD DEVIATIONS AND CORRELATIONS WITH REAL OUTPUT U.S. ECONOMY 1950 : 1–1979 : 2

	Standard Deviations (per cent)	Correlations with Real Output
Output	1.8	—
Total Consumption	1.3	.74
Services	0.7	.62
Non-Durables	1.2	.71
Durables	5.6	.57
Investment Fixed	5.1	.71
Capital Stock		
Durable Mfg.	1.2	−.21
Non-durable Mfg.	0.7	−.24
Inventories	1.7	.51
Hours	2.0	.85
Productivity	1.0	.10

- Parameters - $\{\delta, \beta, \psi\}$ - Chosen to match *long run averages*, then consider cyclical properties...
- Consumption and investment do well
- Model can't get the high volatility in hours

What the model misses

1. *Lacks internal propogation*

   - Only propogation slowing response of Y movements in capital stock
   - These are not large.
   - K is big $\approx 2.5 \times$ GDP, and I is small $\approx 0.11\times$ GDP
   - Would need *very* smooth C, *very* large $\uparrow I$ to move K around a lot

2. *Cannot generate hump-shaped dynamics*

   - In the data, the response of output is also hump-shaped
   - In the model Y increases then monotonically declines
   - Can be corrected with additions to the model
   - But these are mostly unsatisfactory
   - Like to know *behavioural* mechanisms that imply these responses
   - Slow to learn about A_t? Firms don't want to move first?

What the model misses

3. *Maybe not a very satisfying theory of unemployment*

- In a recession, wages are low, so workers choose not to work
- There is no notion of someone being out of work, wanting to work, but not being able to find a job
- We will come back to this with models of *unemployment*

4. *Wages are not that cyclical, while employment is*

- In the model W increases a lot: $\{\uparrow Y, \uparrow C, \uparrow N, \uparrow W\}$
- In the data W are a lot flatter
- Chari, Kehoe, McGrattan (2007) - *Accounting for Business Cycles*
- Find that a shock to labor supply disutility ϕ_t has a lot of explanatory power
- Recession: $\downarrow C_t$, $\uparrow u'(C_t)$ but huge $\downarrow N_t$, while small $\downarrow A_t F_N(K_t, N_t)$

Alternative views of business cycles

- What other mechanisms could generate $\uparrow \{Y, N, C\}$?

- New Keynesian

  - Due to *demand* shocks + Sticky prices

  - Increase in demand for goods. Prices stuck. Given increase in demand produce more. Employ more labor.

- Shocks to profits

  - Profitability increases

    - Decrease in competitiveness, Drop in new firm creation, Change in regulation

  - Firms can make as much profit by producing less

  - Labor falls, output falls, consumption falls

One-period labor supply problem - Increase in τ_N

- Full set of equilibrium conditions

$$
\begin{aligned}
\downarrow AF(\overline{K}, \downarrow N) &= \downarrow C \\
\uparrow AF_N(\overline{K}, \downarrow N) &= \uparrow \frac{W}{P} \\
\downarrow \frac{\downarrow v'(\downarrow N)}{\uparrow u'(\downarrow C)} &= \downarrow \left\{ \Big\downarrow (1 - \uparrow \tau_N) \uparrow \frac{W}{P} \right\}
\end{aligned}
$$

- Household supplies less labor $\downarrow N$
- Output falls $\downarrow Y$, consumption falls $\downarrow C$
- Wage increases due to higher marginal product of labor: $\uparrow W/P$
- Increasing marginal utility of consumption, and decreasing disutility of labor supply would lead the household to want to supply *more* labor.
- For $\downarrow N$, must be that total effect of the tax must be to reduce the total marginal payment to labor.

  W increases less than $(1 - \tau_N)$ increase. $\downarrow Y, \downarrow C, \downarrow N, \uparrow W$