Honors - Economic Analysis III

Lecture 5: Neoclassical model - Gov't and Taxes

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Winter, 2021

This lecture

- Imbed government in neo-classical model
  - Government budget constraint - Taxes, spending, borrowing
  - Term structure of interest rates - Treasury yield curve
  - Theory - Long run capital taxation
  - Theory - Ricardian equivalence
  - Dynamic response to government expenditure and taxes
    - (i) Permanent and temporary ΔG, (ii) Unexpected and anticpated
- Next: Stochastic productivity - RBC model
- PS5 - (i) Responses to different taxes, (ii) Deficit financed corporate tax cut

Government budget constraint

- Government - Finances some *exogenous* stream of spending $\{G_t\}_{t=0}^{\infty}$ E.g. $Q_t = 0.99$

$$G_t + \underbrace{B_t}_{\text{Debt payments}} = \underbrace{Q_t B_{t+1}}_{\text{Debt issuance}} + \underbrace{\tau_t^K R_t K_T + \tau_t^C C_t + \tau_t^T}_{\text{Taxes}} \qquad (*)$$

- Household* - Now pays consumption expenditure tax, capital income tax, lump sum tax:

$$(1+\tau_t^C)C_t + Q_t B_{t+1} + K_{t+1} = W_t N_t + (1-\tau_t^K)R_t K_t + B_t + (1-\delta)K_t - \tau_t^T \quad (**)$$

- Check - When aggregating we obtain the resource constraint. Add $(*)$ and $(**)$:

$$C_t + \Big\{K_{t+1} - (1-\delta)K_t\Big\} + G_t = \Big\{W_t N_t + R_t K_t\Big\}$$

$$C_t + I_t + G_t = Y_t$$

$*$ Already setting profits $\Pi_t = 0$ due to CRS $F(K_t, N_t)$ and competitive factor demand.

Problem - Household

Taking prices $\{W_t, R_t, Q_t\}_{t=0}^{\infty}$ and taxes $\{\tau_t^K, \tau_t^C, \tau_t^T\}_{t=0}^{\infty}$ as given—chooses sequences of $\{C_t, K_{t+1}\}_{t=0}^{\infty}$ to maximize

$$\sum_{t=0}^{\infty} \beta^t u(C_t/N)$$

subject to the series of constraints

$$(1+\tau_t^C)C_t + Q_t B_{t+1} + K_{t+1} \leq W_t N + (1-\tau_t^K)R_t K_t + B_t + (1-\delta)K_t - \tau_t^T$$

and initial conditions

$$K_0 > 0$$

Problem - Household

- First order necessary conditions

$$
\begin{aligned}
K_{t+1}: \quad 0 &= -\lambda_t + \lambda_{t+1}\left[(1-\tau_t^K)R_t + (1-\delta)\right] \\
B_{t+1}: \quad 0 &= -\lambda_t Q_t + \lambda_{t+1} \\
C_t: \quad 0 &= \beta^t u'(C_t) - \lambda_t(1+\tau_t^C)
\end{aligned}
$$

- Euler equation

$$
\begin{aligned}
\frac{u'(C_t)}{1+\tau_t^C} &= \beta \frac{u'(C_{t+1})}{1+\tau_{t+1}^C}\left[(1-\tau_t^K)R_t + (1-\delta)\right] \\
Q_t &= \beta \frac{u'(C_{t+1})/(1+\tau_{t+1}^C)}{u'(C_t)/(1+\tau_t^C)}
\end{aligned}
$$

- Transversality condition

$$
\lim_{T\to\infty} \beta^T u'(C_t)/(1+\tau_T^C) K_{T+1} = 0
$$

Term structure of interest rates

- *Definition* - The *rate of return* on a one-period government bond?

$$r_t = \frac{\text{Payment tomorrow} - \text{Price today}}{\text{Price today}} = \frac{\$1.00 - \$0.95}{\$0.95} = \frac{1 - Q_t}{Q_t}$$

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- Rearrange to reveal the *equilibrium household discount factor* Assume $\tau_t^C = 0$

$$Q_t = \frac{1}{1 + r_t} \quad \rightarrow \quad \frac{1}{1 + r_t} = \frac{\beta u'(C_{t+1})}{u'(C_t)}$$

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- What is the rate of return on a T-period government bond?

$$B_{t+T}: \quad 0 = -\lambda_t Q_{t,T} + \lambda_{t+T}$$

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- Then

$$Q_{t,T} = \frac{\lambda_{t+T}}{\lambda_t} = \frac{\lambda_{t+T}}{\lambda_{t+T-1}} \cdots \frac{\lambda_{t+1}}{\lambda_t} = \frac{\beta u'(C_{t+1})}{u'(C_t)} \cdots \frac{\beta u'(C_{t+T})}{u'(C_{t+T-1})} = \prod_{s=0}^{T} Q_{t+s}$$

Term structure of interest rates

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$$Q_{t,T} = \frac{1}{1 + r_{t,t+T}} = \frac{1}{\prod_{s=0}^{T}(1 + r_s)} = \frac{\beta^T u'(C_T)}{u'(C_t)}$$

Term structure of interest rates

$$Q_{t,T} = \prod_{s=0}^{T} Q_{t+s} \quad \rightarrow \quad \frac{1}{1 + r_{t,t+T}} = \frac{1}{\prod_{s=0}^{T}(1 + r_s)} = \frac{\beta^T u'(C_T)}{u'(C_t)}$$

- Yields - *What* *fixed* *interest rate* $y_{t,T}$ *on a* T*-period government bond would yield the same price?*

$$(1 + y_{t,T})^T = \prod_{s=0}^{T}(1 + r_s)$$

- Suppose consumption was forecast to fall in period T

$$\uparrow Q_{t,T} = \frac{1}{(1 + \downarrow y_{t,T})^T} = \frac{\beta^T \uparrow u'(C_T)}{u'(C_t)}$$

- Demand assets that pay off in period T! Higher price, lower return.

The New York Times

DealBook/ Business & Policy

In the Bond Market, the Economy Is Still Something to Worry About

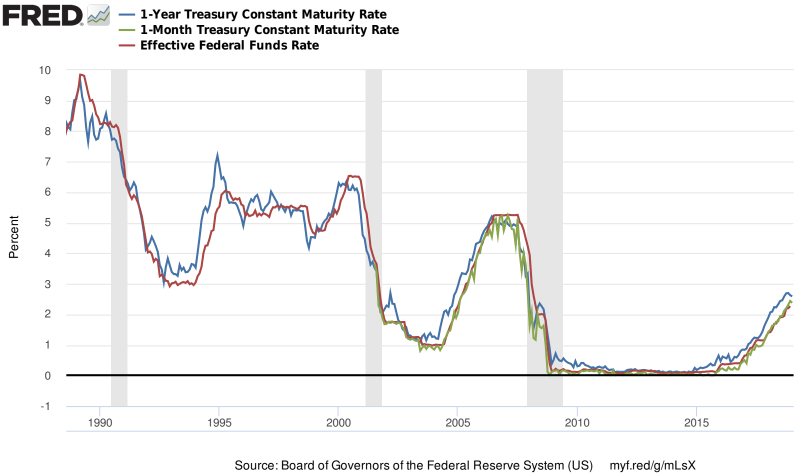
By Stephen Grocer

Jan. 15, 2019

Markets

The U.S. Yield Curve Just Inverted. That's Huge.

The move ushers in fresh questions about the Fed and the economy.


FRED
1-Year Treasury Constant Maturity Rate
1-Month Treasury Constant Maturity Rate
Effective Federal Funds Rate
10
9
8
7
6
5
4
3
2
1
0
-1
Percent
1990
1995
2000
2005
2010
2015
Source: Board of Governors of the Federal Reserve System (US)
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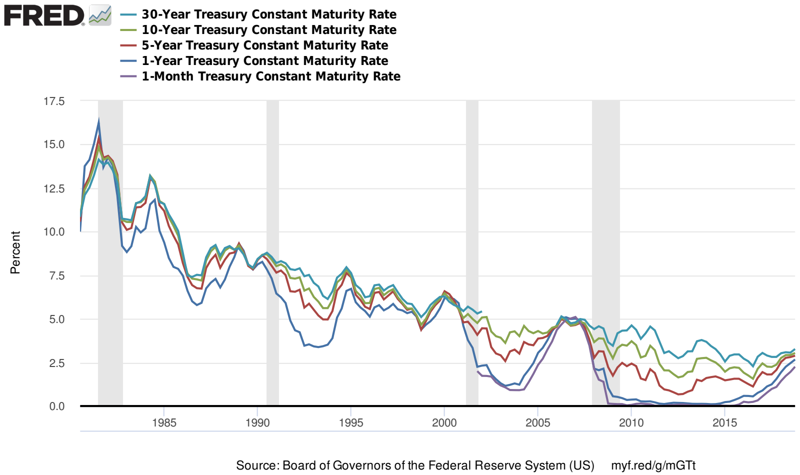
FRED
30-Year Treasury Constant Maturity Rate
10-Year Treasury Constant Maturity Rate
5-Year Treasury Constant Maturity Rate
1-Year Treasury Constant Maturity Rate
1-Month Treasury Constant Maturity Rate
Percent
17.5
15.0
12.5
10.0
7.5
5.0
2.5
0.0
1985
1990
1995
2000
2005
2010
2015
Source: Board of Governors of the Federal Reserve System (US)
myf.red/g/mGTt

Distortionary vs. Non-distortionary taxes

- Given that G_t has to be financed anyway ...
- **Question 1** - Under what sets of taxes do the optimality conditions of the household coincide with the economy *without* taxes?

- Full set of equilibrium conditions

$$
\begin{aligned}
\frac{u'(C_t)}{u'(C_{t+1})} &= \beta \frac{1+\tau_t^C}{1+\tau_{t+1}^C}\left[(1-\tau_t^K)R_t + (1-\delta)\right] \\
Y_t &= C_t + I_t + G_t \\
I_t &= K_{t+1} - (1-\delta)K_t \\
R_t &= F_K(K_t, N) \\
W_t &= F_N(K_t, N) \\
0 &= \lim_{T\to\infty} \beta^T u'(C_t)/(1+\tau_T^C)K_{T+1}
\end{aligned}
$$

Result 1 - Lump-sum τ_t^T are non-distortionary

- Full set of equilibrium conditions

$$
\begin{aligned}
\frac{u'(C_t)}{u'(C_{t+1})} &= \beta\left[R_t + (1-\delta)\right] \\
Y_t &= C_t + I_t + G_t \\
I_t &= K_{t+1} - (1-\delta)K_t \\
R_t &= F_K(K_t, N) \\
W_t &= F_N(K_t, N) \\
0 &= \lim_{T\to\infty} \beta^T u'(C_t) K_{T+1}
\end{aligned}
$$

- Do not see τ_t^T!

Result 2 - Constant τ^C is non-distortionary

- Full set of equilibrium conditions

$$
\begin{aligned}
\frac{u'(C_t)}{u'(C_{t+1})} &= \beta \frac{1+\tau^C}{1+\tau^C} \left[R_t + (1-\delta) \right] \\
Y_t &= C_t + I_t + G_t \\
I_t &= K_{t+1} - (1-\delta) K_t \\
R_t &= F_K(K_t, N) \\
W_t &= F_N(K_t, N) \\
0 &= \lim_{T\to\infty} \beta^T u'(C_t)/(1+\tau^C) K_{T+1}
\end{aligned}
$$

- A constant consumption tax does not distort the economy away from the equilibrium *without* government apart from resource constraint
- Marginal savings decision unaffected since consumption taxed at same rate in all periods leaving the equilibrium rate of return unaffected

Result 2 - Constant τ^C is non-distortionary

- Full set of equilibrium conditions

$$
\begin{aligned}
\frac{u'(C_t)}{u'(C_{t+1})} &= \beta \frac{1+\tau^C}{1+\tau^C}\left[R_t + (1-\delta)\right] \\
Y_t &= C_t + I_t + G_t \\
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\end{aligned}
$$

- A constant consumption tax does not distort the economy away from the equilibrium *without* government apart from resource constraint
- Marginal savings decision unaffected since consumption taxed at same rate in all periods leaving the equilibrium rate of return unaffected
- With *inelastic labor supply* labor taxes also non-distortionary

Result 3 - Constant τ^K *is* distortionary

- Full set of equilibrium conditions

$$
\begin{aligned}
\frac{u'(C_t)}{u'(C_{t+1})} &= \beta\left[(1-\tau^K)R_t + (1-\delta)\right] \\
Y_t &= C_t + I_t + G_t \\
I_t &= K_{t+1} - (1-\delta)K_t \\
0 &= \lim_{T\to\infty} \beta^T u'(C_t)K_{T+1}
\end{aligned}
$$

- Reduces the return on savings
- Reduces equilibrium capital stock

Result 3 - Constant τ^K *is* distortionary

- Full set of equilibrium conditions

$$
\begin{aligned}
\frac{u'(C_t)}{u'(C_{t+1})} &= \beta\left[(1-\tau^K)R_t + (1-\delta)\right] \\
\frac{u'(C_t)}{u'(C_{t+1})} &= \beta\frac{1+\tau_t^C}{1+\tau_{t+1}^C}\left[R_t + (1-\delta)\right]
\end{aligned}
$$

- Reduces the return on savings
- Reduces equilibrium capital stock
- Equivalent to an increasing tax on consumption

$$
\left\{\frac{1}{1-\tau_K}\right\} = \left\{\frac{1+\tau_{t+1}^C}{1+\tau_t^C}\right\} > 1
$$

Ricardean equivalence

- We now understand which taxes distort the path of capital and consumption in the economy
- Suppose the government understands this and so only uses lump-sum taxes

$$G_t + B_t = Q_t B_{t+1} + \tau_t^T$$

- **Question 2**: Does the way G_t is financed affect the economy?
- Consider a government that has to increase government spending for one period

$$G_t = G \quad \text{for all} \quad t \neq T \qquad \text{and in period } T \qquad G_T = G' > G$$

- Many ways to finance this! Two obvious, and topical, ones:

  1. Deficit financed Issue debt in period T, raise taxes in the future: $B_T = G_T$
  2. Tax financed Raise taxes at date T to pay for the spending: $\tau_T^T = G_T$

Ricardean equivalence

- Household budget constraint - Equilibrium returns equalized, so combine K_t and B_t into savings S_t

$$C_t + S_{t+1} = \left(1 + \widetilde{r}_t\right) S_t + W_t N_t - \tau_t^T$$

- Iterate forward by recursively substituting for S_{t+1}, to get *lifetime budget constraint*

$$\sum_{t=0}^{\infty} \frac{C_t}{\prod_{s=0}^{t}(1+\widetilde{r}_s)} = \sum_{t=0}^{\infty} \frac{W_t N_t}{\prod_{s=0}^{t}(1+\widetilde{r}_s)} - \sum_{t=0}^{\infty} \frac{\tau_t^T}{\prod_{s=0}^{t}(1+\widetilde{r}_s)}$$

- Result 1 - *Household only cares about the present discounted value of taxes*
  - Timing of taxes does not affect the household's decision

- Implication - *Household does not care about the composition of S_t*
  - In equilibrium both have the same effective interest rate $\widetilde{r}_t$
  - Household does not treat B_{t+1} differently to K_{t+1}
  - Government debt doesn't *crowd out* public savings.
    S_t same if $B_t > 0$ or $B_t = 0$ (!!!)

Ricardean equivalence

- Government budget constraint

$$G_t + (1 + r_t)B_t = B_{t+1} + \tau_t^T$$

- Iterate forward by recursively substituting for B_{t+1}, to get *lifetime budget constraint*

$$\sum_{t=0}^{\infty} \frac{G_t}{\prod_{s=0}^{t}(1 + r_s)} = \sum_{t=0}^{\infty} \frac{\tau_t^T}{\prod_{s=0}^{t}(1 + r_s)} \quad , \quad \lim_{T\to\infty} \frac{B_{t+T}}{\prod_{s=0}^{T}(1 + r_s)} = 0$$

- Result 2 - Present discounted value of taxes must equal PDV of government expenditure

- Combine 1 & 2 - Households are only affected by the PDV of government expenditure. How it is financed is irrelevant! This is *Ricardean equivalence*

Ricardean equivalence - Assumptions

Consider a policy of a tax-cut today $\tau_t < 0$, followed by a tax-increase in the future $\tau_T > 0$, with $G = 0$ for all t. Household saves and pays back in future.

1. *Credit markets* - Credit markets are perfect
   - Ricardean household saves today and pays back later.
   - A *constrained household* might consume some now and pay back through later income

2. *Information* - Household will be alive when/understands that future tax increase
   - Ignoring future tax increases would lead to increased spending

3. *Symmetry* - Households all experience the tax-cut in the same way
   - Tax cuts can *redistribute* wealth when wealth is held unequally. Maybe not so important for *aggregate* outcomes. But maybe we care about the distribution?

4. *Non-distorting tax* - Taxes are lump sum
   - In reality taxes are distorting. In response to a *labor tax cut* our Ricardean household will *work more today* and *work less tomorrow*.

Dynamics

- *Ricardean equivalence* makes our life easy, we only have to consider the dynamics of g_t to solve the household's path of consumption and savings
- Let's start with the economy in steady state with some initial g

$$1 = \beta[f'(\bar{k}) + (1-\delta)] = \beta(1 + \widetilde{r}(\bar{k}))$$

$$\bar{c} = f(\bar{k}) - \delta\bar{k} - g$$

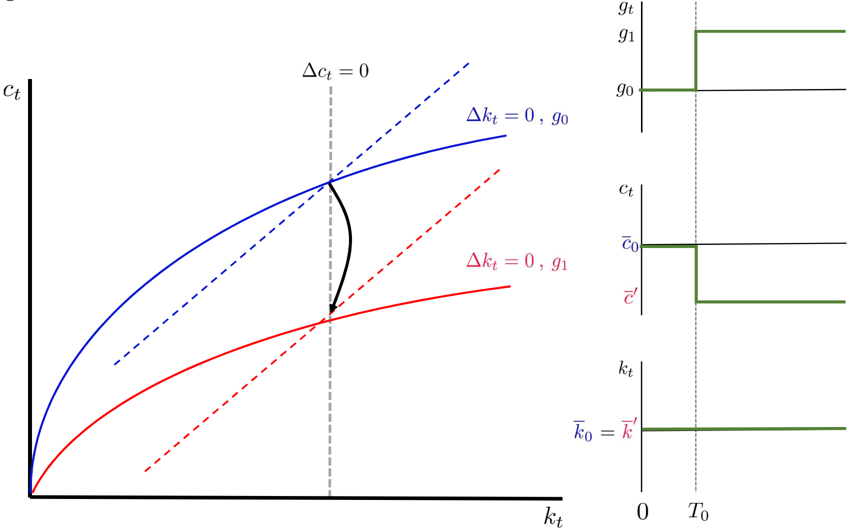
- Consider four different changes in government spending $g' > g$

1. Changes at date 0. Unexpected 'shocks'
   - Permanent increase in g from period T_0 onwards
   - Transitory increase in g from period T_0 to T_1

2. Increases at date T_0, but anticipated from date 0
   - Permanent increase in g from period T_0 onwards
   - Transitory increase in g from period T_0 to T_1

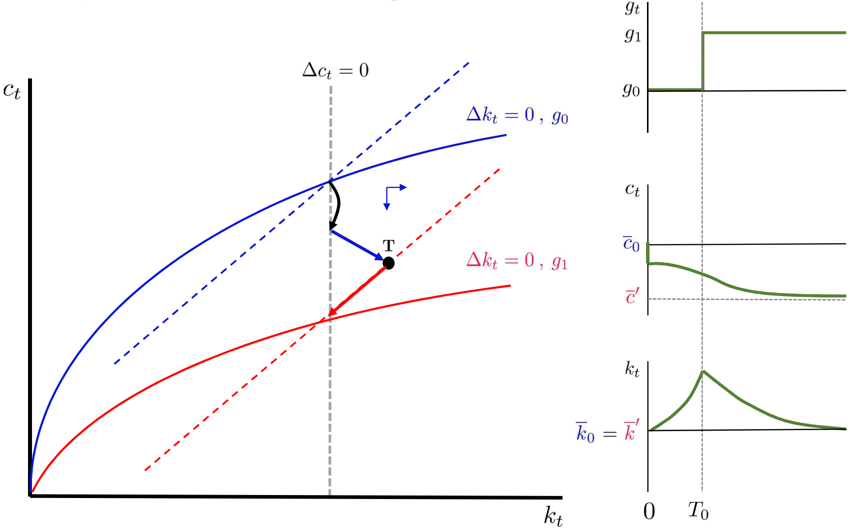
Dynamics - $\uparrow g$ - Unexpected - Permanent

$\downarrow c_0$, no change in the interest rate.



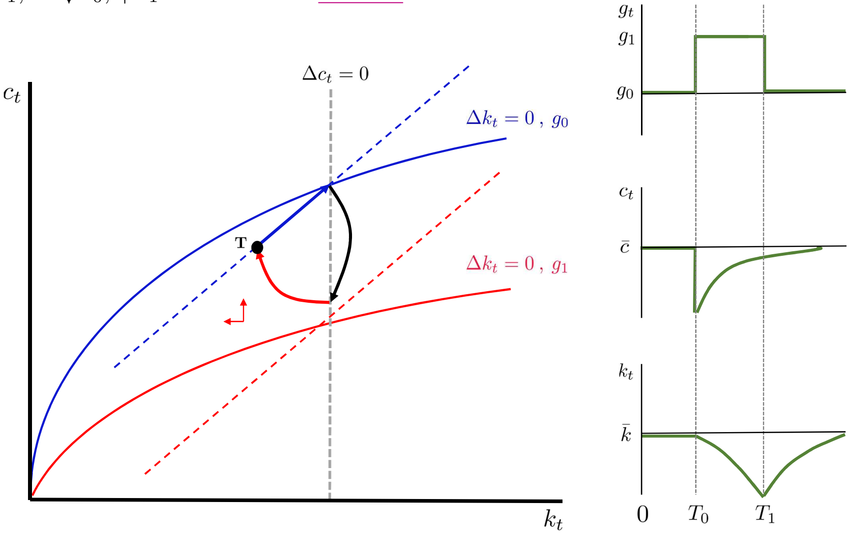
Dynamics - $\uparrow g$ - Anticipated - Permanent

Fall in interest rate $\downarrow R_1$, household cuts consumption. Interest rate <u>decrease</u>



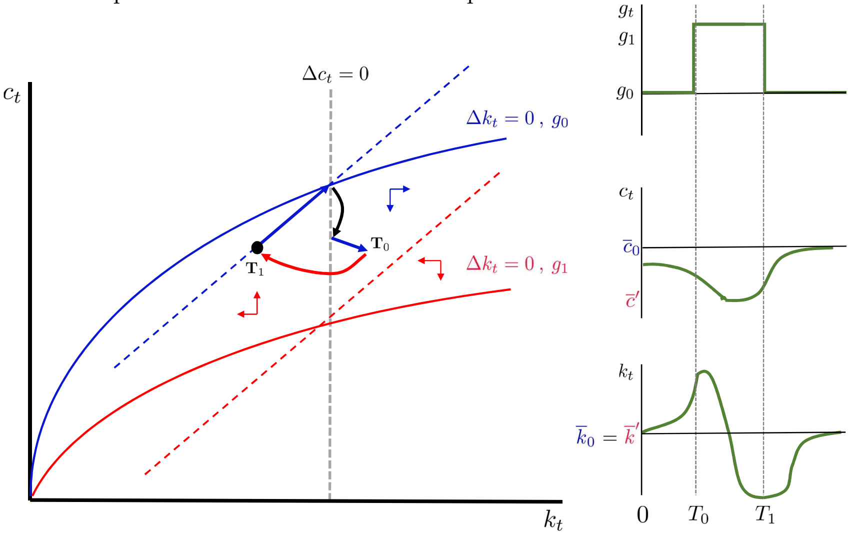
Dynamics - $\uparrow g$ - Unexpected - Transitory

Increase in R_1, so $\downarrow c_0$, $\uparrow c_1$. Interest rate <u>increase</u>



Dynamics - $\uparrow g$ - Anticipated - Transitory

$\downarrow c_0$ and accumulate capital to smooth effect on consumption.



Dynamics - Anticipated transitory increase

- Try yourself! I'll put up solution in slides next week
- Use what we have discussed here
- What is steady state capital and consumption be in the long run?
- What must happen to consumption on impact?
- Where must the economy be when G' drops back down to G?
- What's the relative rate of change in the economy along the path?
- *In all cases give an intuitive explanation of the dynamics in terms of the equilibrium behaviour of households in the economy and how they are responding to prices*

END