Honors - Economic Analysis III

Lecture 8: Labor supply

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This lecture

- Simple one period problem in *partial equilibrium*
  - Income and substitution effects
- Endogenous labor supply in neoclassical model
  - Taxes on labor supply
- *General equilibrium* response to changes in A, G
- Next - Labor supply + Stochastic neoclassical $\rightarrow$ RBC model
- PS5 - (1) Static labor supply, (2) Dynamic labor supply

One-period labor supply problem - P.E.

- The main endowment that we all sell in the market place is our labor!

- *Endowment* - Individual endowed with $\overline{L}$ units of labor / time

- Choose the amount of labor to supply $N \in [0, \overline{L}]$

- Whatever is left is *leisure* $L = \overline{L} - N$

- Preferences:

$$U(C, L), \quad U_L > 0, \quad U_{LL} < 0, \quad \lim_{L \to 0} U_L = \infty, \quad \lim_{L \to \overline{L}} U_L = 0$$

- *Leisure is a normal good*

One-period labor supply problem - P.E.

- Preferences:

$$U(C, L), \quad U_L > 0, \quad U_L L < 0, \quad \lim_{L \to 0} U_L = \infty, \quad \lim_{L \to \overline{N}} U_L = 0$$

- Problem

$$\max_{C,N} \; U(C, \overline{L} - N)$$

subject to

$$PC \leq WN$$

$$\underbrace{PC + WL}_{\text{Buy consumption and leisure}} \leq W\overline{L} \quad \rightarrow \quad \uparrow W \text{ increases (i) income, (ii) cost of leisure}$$

One-period labor supply problem - P.E.

- Lagrangian

$$\mathcal{L} = U(C, \overline{L} - N) + \lambda \left[WN - PC \right]$$

- First order conditions

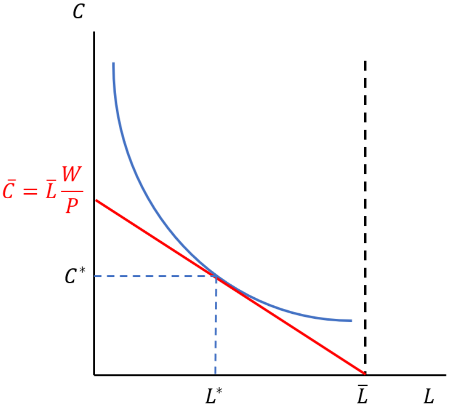
$$\begin{aligned} N: \qquad 0 &= -U_L(C, \overline{L} - N) + \lambda W \\ C: \qquad 0 &= U_C(C, \overline{L} - N) - \lambda P \end{aligned}$$

- Combined - Intratemporal labor supply condition

$$\frac{U_L(C, \overline{L} - N)}{U_C(C, \overline{L} - N)} = \frac{W}{P}$$

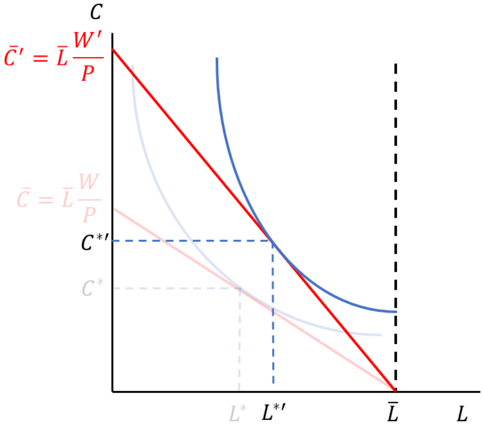
Marginal rate of substitution between leisure and consumption =
Relative price of leisure to consumption

One-period labor supply problem - P.E.



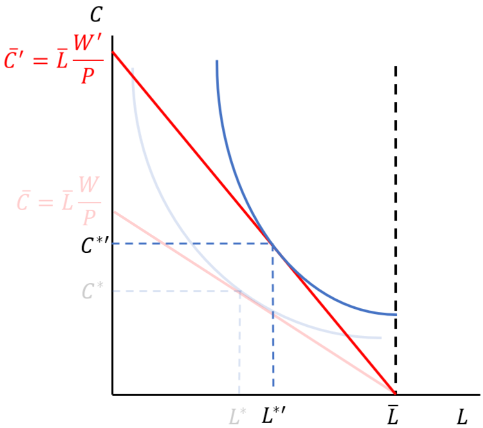
$$PC = WN \quad , \quad \frac{U_L(C, \overline{L} - N)}{U_C(C, \overline{L} - N)} = \frac{W}{P}$$

One-period labor supply problem - P.E.



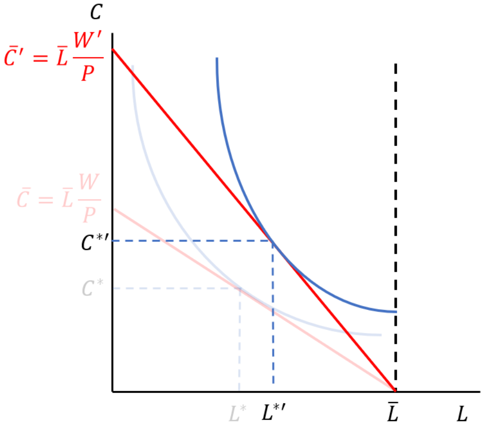
$$PC = W'N \quad , \quad \frac{U_L(C, \overline{L} - N)}{U_C(C, \overline{L} - N)} = \frac{W'}{P}$$

One-period labor supply problem - P.E.



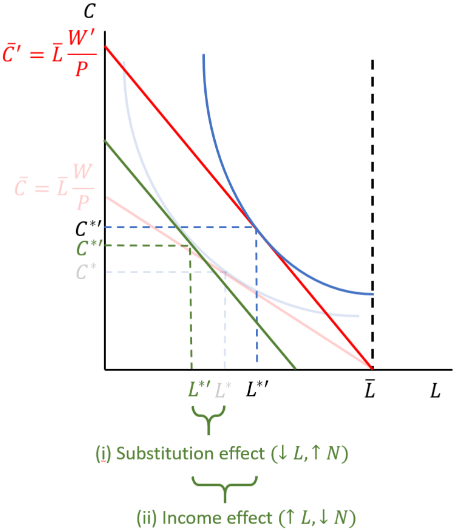
$$PC = W'\left(\overline{L} - L\right) \quad , \quad \frac{U_L(C, \overline{L} - N)}{U_C(C, \overline{L} - N)} = \frac{W'}{P}$$

One-period labor supply problem - P.E.



$$PC + W'L = W'\overline{L} \quad , \quad \frac{U_L(C, \overline{L} - N)}{U_C(C, \overline{L} - N)} = \frac{W'}{P}$$

One-period labor supply problem - P.E.



One-period labor supply problem - G.E.

- Fix the economy's supply of capital at $\overline{K}$

- Competitive firm with constant returns to scale production technology.

$$Y = AF(K, N)$$

- *Question:* What is the effect of an increase in productivity A on labor supply in general equilibrium?
- Profit maximization

$$\max_{N,K} PAF(K, N) - WN - RK$$

- Equilibrium wage: (i) First order condition for N, (ii) impose $K = \overline{K}$

$$\frac{W}{P} = AF_N\left(\overline{K}, N\right)$$

One-period labor supply problem - G.E.

- General equilibrium: $Y = C$, $PC = WN + R\overline{K} + \Pi$
- Can draw this out as a production possibility frontier
- Given L how much C can be produced?

$$C = F(\overline{K}, \overline{L} - L)$$

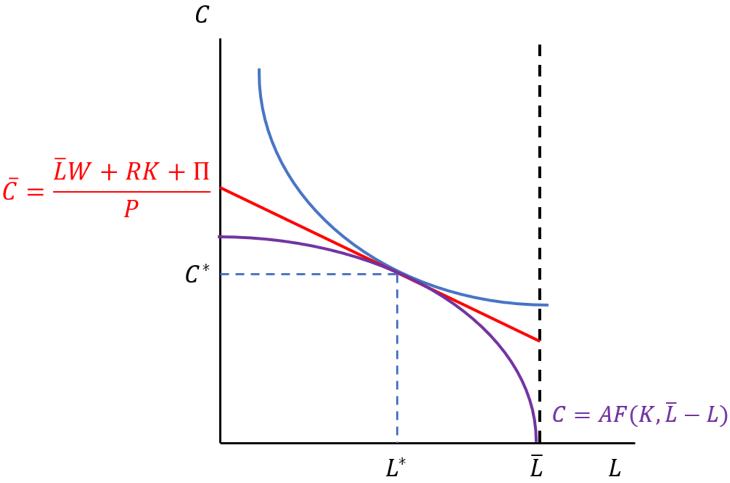
- The slope of this line is given by the marginal product of labor:

$$\frac{dC}{dL} = -AF_N(\overline{K}, N)$$

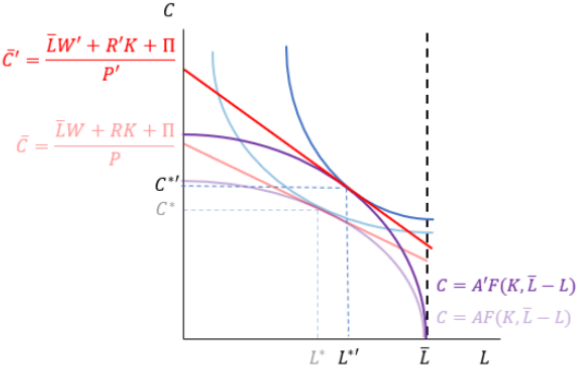
- General equibrium

$$\underbrace{AF_N(\overline{K}, N)}_{\text{Marginal product}} = \underbrace{\frac{W}{P}}_{\text{Wage}} = \underbrace{\frac{U_L(C, \overline{L} - N)}{U_C(C, \overline{L} - N)}}_{\text{Marginal rate of substitution}}$$

One-period labor supply problem

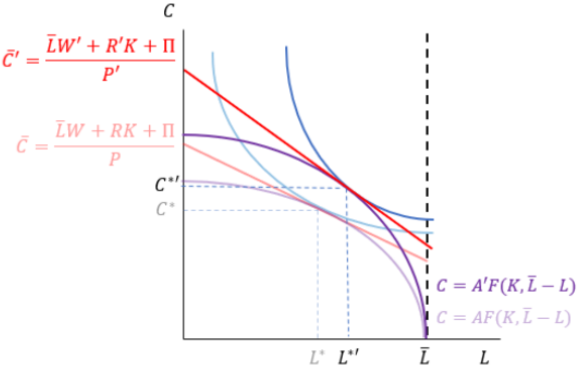


One-period labor supply problem - $\uparrow A$



- Effect of A on N depends on Income vs. Substitution effect
- If Income effect is large $corr(A, N) < 0$ (pictured)
- If Substitution effect is large $corr(A, N) > 0$

One-period labor supply problem - $\uparrow A$



- **Solow model** - In the long-run increases in A generate growth
- **Fact** - Data from most countries show N approximately constant
- **Implication** - Income and substitution effects seem to offset in long-run

One-period labor supply problem - Taxes

- Problem

$$\max_{C,N} \; U(C, \overline{L} - N)$$

subject to

$$(1 + \tau_C)PC \leq (1 - \tau_N)WN$$

- First order condition

$$\frac{U_L(C, \overline{L} - N)}{U_C(C, \overline{L} - N)} = \frac{1 - \tau_N}{1 + \tau_C} \frac{W}{P}$$

- Result - Constant labor and consumption taxes will have same affect on equilibrium conditions. τ_N reduces the value of labor supply, τ_C increases the cost of the stuff that labor buys.

One-period labor supply problem - Comparative statics

Take the G.E. one period labor supply problem and consider three different comparative statics

1. An increase in government spending G

2. An increase in productivity A

3. An increase in labor income taxes τ_N

Moving towards understanding *business cycles* we will keep in mind that in the data the following all seem to go up in a boom and down in a recession

Output (Y), Consumption (C), Hours (N), Wages (W/P)

One-period labor supply problem - Increase in G

- Full set of equilibrium conditions

$$\begin{aligned} AF(\overline{K}, N) &= C + G \\ AF_N(\overline{K}, N) &= \frac{W}{P} \\ \frac{U_L(C, \overline{L} - N)}{U_C(C, \overline{L} - N)} &= \frac{W}{P} \end{aligned}$$

- Suppose that preferences are such that

$$U(C, L) = u(C) - v(\overline{L} - L)$$

$$U(C, N) = u(C) - v(N)$$

One-period labor supply problem - Increase in G

- Full set of equilibrium conditions

$$
\begin{aligned}
AF(\overline{K}, N) &= C + G \\
AF_N(\overline{K}, N) &= \frac{W}{P} \\
\frac{v'(N)}{u'(C)} &= \frac{W}{P}
\end{aligned}
$$

- Suppose that preferences are such that

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$$U(C, N) = u(C) - v(N)$$

One-period labor supply problem - Increase in G

- Full set of equilibrium conditions

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\begin{aligned}
AF(\overline{K}, N) &= C + \uparrow G \\
AF_N(\overline{K}, N) &= \frac{W}{P} \\
\frac{v'(N)}{u'(C)} &= \frac{W}{P}
\end{aligned}
$$

One-period labor supply problem - Increase in G

- Full set of equilibrium conditions

$$\begin{aligned}
\downarrow AF(\overline{K}, \downarrow N) &= \downarrow\downarrow C + \uparrow G \\
\uparrow AF_N(\overline{K}, \downarrow N) &= \uparrow \frac{W}{P} \\
\uparrow \frac{\downarrow v'(\downarrow N)}{\downarrow u'(\uparrow C)} &= \uparrow \frac{W}{P}
\end{aligned}$$

- Suppose output **decreases**, then C must have decreased and N decreased to produce less
- Increases marginal product of labor. Wage must increase.
- To supply *less labor* at a *higher wage*, $u'(C)$ must have really fallen.
- This occurs when consumption *increases*.

  Contradiction
- If I'm not going to bother working when the wage is increasing, my marginal utility of consumption *must* have tanked!

One-period labor supply problem - Increase in G

- Full set of equilibrium conditions

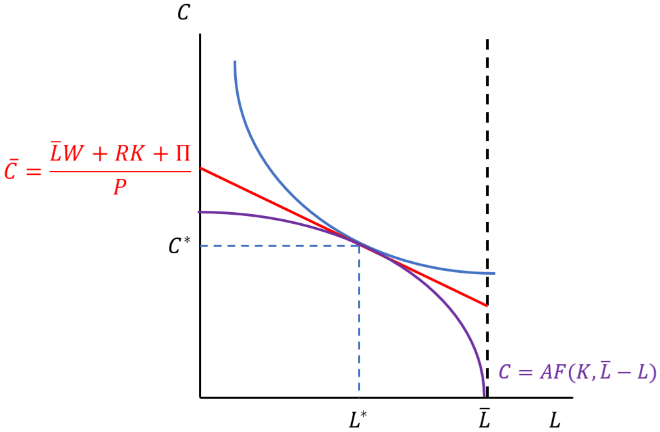
$$\begin{aligned} \uparrow AF(\overline{K}, \uparrow N) &= \downarrow C + \uparrow G \\ \downarrow AF_N(\overline{K}, \uparrow N) &= \downarrow \frac{W}{P} \\ \downarrow \frac{\uparrow v'(N)}{\uparrow u'(\downarrow C)} &= \downarrow \frac{W}{P} \end{aligned}$$

- Suppose output **increases**, then N must increase to produce more
- Reduces marginal product of labor. Wage must fall.
- To supply more labor when wages are falling, $u'(C)$ must have increased
- This will be the case if consumption is lower

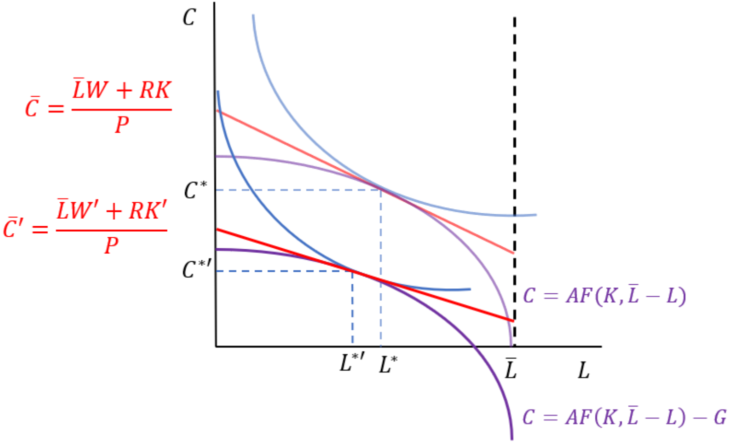
$$\Big\uparrow Y = \downarrow C + \Bigg\uparrow G$$

- Result - An increase in G causes $\uparrow Y$, $\uparrow N$, $\downarrow C$, $\downarrow W/P$ ... not like the data

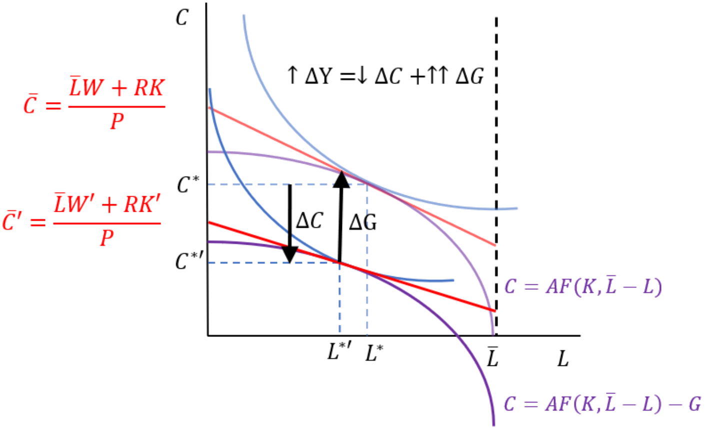
One-period labor supply problem



One-period labor supply problem



One-period labor supply problem



One-period labor supply problem - Increase in A

- Full set of equilibrium conditions

$$
\begin{aligned}
\uparrow AF(\overline{K}, \updownarrow N) &= \uparrow C + G \\
\uparrow AF_N(\overline{K}, \updownarrow N) &= \uparrow \frac{W}{P} \\
\frac{\updownarrow v'(N)}{\downarrow u'(\uparrow C)} &= \uparrow \frac{W}{P}
\end{aligned}
$$

- Output increases, therefore C increases (recall production possibility frontier)
- Marginal utility of consumption decreases $\downarrow u'(C)$, so for any (W/P) supply less labor
- Wage must go up: (i) Left shift in labor supply, (ii) Right shift in labor demand
- Ambiguous effect on N
  - Increase in $W \rightarrow \downarrow N$ (Income effect) if *large* fall in $u'(C)$
  - Increase in $W \rightarrow \uparrow N$ (Substitution effect) if *small* fall in $u'(C)$
- Consumption and output increase

$$\uparrow Y = \uparrow C + G$$

- Result - An increase in A causes $\uparrow Y$, $\updownarrow N$, $\uparrow C$, $\uparrow W/P$ ... potentially like the data

One-period labor supply problem - Increase in τ_N

- Full set of equilibrium conditions

$$\begin{aligned}
\downarrow AF(\overline{K}, \downarrow N) &= \downarrow C + G \\
\uparrow AF_N(\overline{K}, \downarrow N) &= \uparrow \frac{W}{P} \\
\color{red}{\downarrow} \frac{\downarrow v'(\downarrow N)}{\uparrow u'(\downarrow C)} &= \color{red}{\downarrow} \left\{ \Big\downarrow (1 - \color{blue}{\uparrow \tau_N}) \uparrow \frac{W}{P} \right\}
\end{aligned}$$

- Must be the case that the household supplies less labor $\downarrow N$
- Output falls $\downarrow Y$, consumption falls $\downarrow C$
- Wage increases due to higher marginal product of labor: $\uparrow W/P$
- Increasing marginal utility of consumption, and decreasing disutility of labor supply would lead the household to want to supply *more* labor.
- For $\downarrow N$, must be that total effect of the tax must be to reduce the total marginal payment to labor.

  W increases less than $(1 - \tau_N)$ increase. $\color{red}{\downarrow Y, \downarrow C, \downarrow N, \uparrow W}$

One-period labor supply problem - Basu Pindyk

- Add their uncertainty example with flexible P, and then do it again with markups